Earth Observation and Geomatics Engineering 5(1) (2021) 12-17

EARTH OBSERVATION AND GEOMATICS ENGINEERING

website: https://eoge.ut.ac.ir

On the covariant formulation of geoid: the case for Kerr spacetime

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Article history:
Received: 26 March 2020, Received in revised form: 29 January 2021, Accepted: 5 February 2021

ABSTRACT

A compatible theory of relativistic geodesy is quite necessary for future development of any formulation of the subject. In this paper, following recent studies which invoke an idea of relativistic geoid, i.e. the chronometric definition implied by the gravitational redshift, we revisit known calculations on some standard spacetimes. Among all, the notion of frame dragging in the Kerr spacetime and its relationship to the rotative observers needed for the consistent definition of the geoid is analyzed.

KEYWORDS

Physical geodesy
Geoid
Relativity.

1. Introduction

One of important concepts in physical geodesy is geoid which is an equipotential surface of the gravitational potential. The classical theory of geodesy uses the Newtonian definition of potential while, up to now, the correct theory of gravitation is that of Einstein's. Therefore, regardless of applicability of the theory, it sounds acceptable to develop the relativistic theory of geodesy consisting of relativistic geoid, gravimetry, gradiometry and so on. Another motivation for inclusion of the general theory of relativity in our thoughts around geodesy is related to the ever-increasing precision of the measurements. By now, it is quite possible to measure relative differential frequencies of order 10^{-18} using optical clocks (Muller et al. 2018; Denker et al., 2017) . The stability of such clocks is the more important issue which is as much as 1 part in 10^{15}. To compare this numbers with the frequency shift due to the special relativistic Doppler effect, let's see an example. Suppose a GPS satellite orbiting with the velocity $4000\ m/s$ relative to the geocenter. The Lorentz factor amounts to $\sqrt{1-v^2/c^2} \cong 1-8.5\times10^{-11}$, for which, if we sum over a day, will end up to $1-7.3\times10^{-6}$, i.e. 7.3 microseconds (μS). This is remarkable relative to precision and working frequency of GPS satellites which is almost 10^{-8} or $0.01\ \mu S$ (Muller et al., 2008).

The general relativistic contribution to frequency shift is much more strong. Contrary to Newtonian theory of gravity, in the framework of general relativity the time is not absolute from the point of view of different observers and it depends on the distance the observer is located from geocenter, i.e. the center of the source of the gravitational field. According to the principle of equivalence, the closer the observer is to the source of gravity the slower the time passes. As we will see, this shift of time or frequency (usually called redshift) is responsible for another difference in the frequency of the received signals according to the following correspondence (Delva et al., 2017):

$$1\ cm \leftrightarrow \frac{\delta\nu}{\nu} \approx 10^{-18} \leftrightarrow \delta W \sim 0.1\,\frac{m^2}{s^2} \qquad (1)$$

where W and ν are gravitational potential and frequency respectively. This is exactly what we want to define a geoid. The idea reads:

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DOI: 10.22059/eoge.2021.324997.1095

Since the redshift is directly related to the height difference, it is possible to find the difference in the height of two points in a gravitational field having already the corresponding frequency differences.

In other words, the above result enable us to measure the height directly. The first who came to this idea and defined the relativistic geoid was Bjerhammar (Bjerhammar, 1985,1986).

"The relativistic geoid is the surface where precise clocks run with the same speed and the surface is nearest to mean sea level."

This definition, however, is somehow operational as the gravitational potential, and consequently the related redshift, needs first to be defined precisely. Many researches have been conducted in pursuit of full relativistic treatment of the geoid, gravimetry and gradiometry. To sketch the gravitational potential, some authors have used the post-Newtonian approximation of the spacetime metric, see for instance (Muller, 2008; Kopejkin, 2018; Kopejkin, 2016;Kopejkin, 2016-2;Soffel, 2016;Kopejkin, 2015), and others do it while keeping the full covariance of the theory (Philipp, 2020;Delva, 2017;Philipp, 2017;Oltean, 2016). Although the post-Newtonian method is not an approximating approach, we follow the second approach as it is more compatible to the theory of observers in curved spacetime.

Following recent developments which try to give a precise definition of the gravitational potential, see for instance (Philipp et al., 2020; Soffel et al., 2016), we calculate the relativistic correction to the gravitational potential for the case of the Schwarzschild and Kerr spacetimes using a more reliable theory for rigidly corotative observers.

2. The redshift

In this section we review some basic concepts related to general relativity and one of its most popular consequences, i.e. the time dilation or what traditionally called redshift. The spacetime in the Einstein theory of gravity is described by a typical metric of the form

$$ds^2 = c^2 d\tau^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad \mu,\nu = 0,1,2,3,4. \tag{2}$$

where τ is the proper time measured by an observer located at point $(x^0, x^1, x^2, x^3) = (t, r, \theta, \phi)$. The time variable t is called coordinate time. The metric components in (2) satisfy the Einstein field equations (EFEs)

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \tag{3}$$

At the left hand side of (3), $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is called the Einstein tensor and on the other side, $T_{\mu\nu}$ is the stress-energy-momentum tensor which demonstrates the matter content of the source of gravity. $R_{\mu\nu} = R_{\alpha\mu\nu}^{\alpha}$ is the Ricci tensor obtained by contraction of the Riemann's tensor. The Ricci scalar $R = g^{\nu\alpha} R_{\nu\alpha}$ is also the contraction of the Ricci tensor. In its own turn, the Riemann's tensor is given by:

$$\begin{aligned} R_{\mu\nu\alpha}{}^{\beta} &= \partial_\mu \Gamma_{\nu\alpha}{}^{\beta} - \partial_\nu \Gamma_{\mu\alpha}{}^{\beta} + \Gamma_{\mu\rho}{}^{\beta} \Gamma_{\nu\alpha}{}^{\rho} - \Gamma_{\nu\rho}{}^{\beta} \Gamma_{\mu\alpha}{}^{\rho}, \\ \Gamma_{\mu\nu}^{\sigma} &= 12 g^{\sigma\rho} (\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}) \end{aligned} \tag{4}$$

where $\Gamma_{\mu\alpha}{}^{\beta}$ is the Levi–Civita connection. The most well-known solution to EFEs derived first by Carl Schwarzschild is described by the line element

$$ds^2 = -(1 - 2GMc^2 r) c^2 dt^2 + (1 - 2GMc^2 r)^{-1} dr^2 + r^2 d\Omega^2 \tag{5}$$

in which $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $m = GM/c^2$ is known as the gravitational mass of the source. For a typical mass $M = 5.972 \times 10^{24}\ kg$ of the Earth it amounts to $m \simeq 4.4 \times 10^{-3} cm$.

To discuss about the redshift, suppose two observers located at the same ϕ and θ but at distinct radial coordinates r, say at r_2 and r_1. If the observer 1 send two crests of the light ray at two different proper times shown by $\Delta\tau_1$, using (5), she/he can find the following relationship between her/his coordinate and proper time

$$\Delta t_1 = (1 - 2mr_1)^{-1/2} \Delta\tau_1. \tag{6}$$

Note that she/he has been fixed at its own location shown by (r_1, θ_1, ϕ_1) hence $\Delta r_1 = \Delta\theta_1 = \Delta\phi_1$. The observer 2 receivess the two signals by the spacetime distance $\Delta\tau_2$ and separately finds

$$\Delta t_2 = (1 - 2mr_2)^{-1/2} \Delta\tau_2. \tag{7}$$

We set $\Delta t_1 = \Delta t_2$ in both cases as the coordinate time does not change along the null geodesic traveled by photons, see section 4.3 of (Foster, 2006). By combining (5) and (6) we arrive at (Carroll, 1997)

$$\Delta\tau_2 = (1 - 2m/r_2 1 - 2m/r_1)^{1/2} \Delta\tau_1. \tag{8}$$

This equation shows that the proper time measured by two distinct observers between two emissions of the light source is different. In other words, the time between two clock ticks, taking the ticks as emitting signals, is different from point of view of different observers. This is called gravitational redshift and should be distinguished from the Doppler shift induced by relative velocity of two observers.

One can show, along the same steps as the above, that for a general static spacetime we have

$$\nu_1 = \left(\frac{g_{00}(X_2)}{g_{00}(X_1)}\right)^{1/2} \nu_2, \tag{9}$$

in which X_1 and X_2 are the locations of the observers (clocks). Note that we have used the equivalent clock frequencies $\Delta\nu$ instead of the corresponding proper time. By expansion of (9) up to first order in terms of $1/c^2$ we arrive at

$$\frac{\Delta\nu}{\nu}=\frac{\nu_1-\nu_2}{\nu_1}=\left(\frac{m}{r_1}-\frac{m}{r_2}\right)+O(1/c^4). \quad (10)$$

The two terms at the right-hand side of equation (10) are nothing but the Newtonian gravitational potential. Thus, we see that the difference in the gravitational potential is directly related to the shift on the observed frequency of the clocks as follows

$$\frac{\Delta\nu}{\nu}=\frac{W(X_1)-W(X_2)}{c^2}+O(1/c^4). \quad (11)$$

In brief, a surface is an equipotential surface, i.e. the geoid, if the difference in the observed frequency of the clocks between any two points on the surface vanishes.

3. The Newtonian limit of general relativity

The appearance of the gravitational potential at the right-hand side of (10) does not happened by accident. In this section we investigate the relationship between metric components and the Newtonian potential. To do so, we need to review first the Newtonian limit of general relativity. Before that, a point is worthy of notice. Since we supposed that our observers are fixed at some spacetime point, the analysis given in previous section seems not to be satisfactory. Our goal is to clarify the role of moving observers to the amount of the observed redshift, i.e. to put some corrections on equation (10).

The EFEs on equation (3) are nonlinear. If we put some ansatz on it, an standard linearized theory can be obtained and the Newton's theory of gravity recovered. Intuitively, there must be some familiarities between EFEs and Poisson's equation of Newtonian theory. To transform EFEs into its Newtonian counterpart, we assume that the gravitational field under consideration is weak in the sense that metric is given by $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu},\ \ h_{\mu\nu}<<1$, where $\eta_{\mu\nu}=diag(-1,1,1,1)$ is the metric of flat spacetime. Thus, the metric is almost that of flat spacetime plus a perturbation $h_{\mu\nu}$. The 00-component of the EFEs then reads

$$R_{00}=\kappa(T_{00}-\tfrac{1}{2}Tg_{00}),\quad \kappa=\frac{8\pi G}{c^4}. \quad (12)$$

We assume also that the stress tensor for the source of gravity is demonstrated by a perfect fluid given by $T_{\mu\nu}=\rho v_\mu v_\nu$ in which ρ is the density of mass and $v^\mu=c(1,0,0,0)$ the corresponding velocity four-vector measured by a co-moving observer. Since in the Newotonian limit we have $|\vec{v}|<<c$, the above stress tensor is eligible and $T_{00}=\rho v_0 v_0\simeq\rho,\ T=\rho c^2$. Substituting all this back into (12), it recasts into:

$$R_{00}=\tfrac{1}{2}\kappa\rho c^2. \quad (13)$$

At the left-hand side, to find R_{00} in terms of metric components, it suffices to substitute the weak metric into (4) while keeping only the terms up to first order perturbations in terms of $h_{\mu\nu}$. Thus we do as follows.

Using the criteria $g_{\mu\nu}=\eta_{\mu\nu}+h_{\mu\nu},\ \ h_{\mu\nu}<<1$, we have

$$g^{\mu\nu}=\eta^{\mu\nu}-h^{\mu\nu}+O(h^2),\quad \partial_\lambda g_{\mu\nu}=\partial_\lambda h_{\mu\nu}. \quad (14)$$

and from

$$\Gamma^\sigma_{\mu\nu}=12g^{\sigma\rho}(\partial_\mu g_{\nu\rho}+\partial_\nu g_{\rho\mu}-\partial_\rho g_{\mu\nu}) \quad (15)$$

we find

$$\Gamma^\sigma_{\mu\nu}=12\eta^{\sigma\rho}(\partial_\mu h_{\nu\rho}+\partial_\nu h_{\rho\mu}-\partial_\rho h_{\mu\nu}) \quad (16)$$

in which second order perturbation terms have been neglected, e.g. $\Gamma^\sigma_{\mu\nu}\propto O(h_{\mu\nu})$ and $\Gamma_{\nu\rho}^{\ \ \beta}\Gamma_{\mu\alpha}^{\ \ \rho}=O(h^2_{\mu\nu})\equiv 0$. Therefore, using equation (4) we have

$$R^\mu_{\ \nu\alpha\beta}=\partial_\alpha\Gamma^\mu_{\ \nu\beta}-\partial_\beta\Gamma^\mu_{\ \nu\alpha}+O(h^2_{\mu\nu}), \quad (17)$$

which in turn yields

$$R_{\nu\beta}=R^\mu_{\ \nu\mu\beta}=\partial_\mu\Gamma^\mu_{\ \nu\beta}-\partial_\beta\Gamma^\mu_{\ \nu\mu}, \quad (18)$$

and

$$R_{00}=\partial_\mu\Gamma^\mu_{00}-\partial_0\Gamma^\mu_{\ 0\mu}. \quad (19)$$

As the spacetime supposed to be static, the second term which consists of derivative with respect to time vanishes and it founds that

$$R_{00}=\partial_\mu\Gamma^\mu_{00}. \quad (20)$$

The term Γ^μ_{00} can be easily found from (16) as follows

$$\begin{aligned}\Gamma^\mu_{00}&=\tfrac{1}{2}\eta^{\mu\lambda}(\partial_0 h_{\lambda 0}+\partial_0 h_{\lambda 0}-\partial_\lambda h_{00})\\&=-\tfrac{1}{2}\eta^{\mu\lambda}\partial_\lambda h_{00}\\&=-\tfrac{1}{2}\eta^{\lambda i}\partial_i h_{00}\end{aligned} \quad (21)$$

where we have used again the time independency of the metric in the last line. Note that $i=1,2,3$ and $\mu=0,1,2,3$. From (21) we then find

$$\Gamma^0_{00}=0,\quad \Gamma^j_{00}=-\tfrac{1}{2}\eta^{ji}\partial_i h_{00}. \quad (22)$$

Finally, equation (20) should be written as

$$R_{00} = -\frac{1}{2}\eta^{ji}\partial_j\partial_i h_{00} = -\frac{1}{2}\nabla^2 h_{00}. \qquad (23)$$

which in turn gives

$$-\frac{1}{2}\delta^{ij}\partial_i\partial_j h_{00} = \frac{1}{2}\kappa\rho c^2. \qquad (25)$$

Using the equality $\delta^{ij}\partial_i\partial_j \equiv \nabla^2$ we recover the Poisson's equation under the condition that $h_{00} = \frac{2}{c^2}W$, where W being the gravitational potential.

The above analysis shows that the component $-g_{00} = 1 + h_{00}$ plays an important role for the determination of the Newtonian potential from EFEs at the weak field limit.

4. The rotative observers

In previous section we review some basic concepts of general theory of relativity and its capabilities to model the gravitational field. Obviously one may asks whether the above analysis could be generalized to taking also the moving observers into account. Actually, the Earth is rotating and all observers in the laboratories corotates with it. Some authors impose rotations on the Schwarzchild spacetime adhoc to model the Earth's rotation. This adhoc rotations are traditionally composed of a simple transformation of the form

$$\overleftarrow{X} \rightarrow \overleftarrow{\mathbf{X'}} = \overleftarrow{X} - \overleftarrow{\boldsymbol{\omega}} t. \qquad (26)$$

This type of transformation eventually, after going to the Newotonian limit, modifies the Newotonian potential as $W(x) \rightarrow W(X) - \frac{a}{2}r^2 sin^2\theta$, just like the one which always done in the textbooks of classical geodesy. This enteric the community to put on the desk the relativistic theory of corrotative observers to show the natural appearance of the terms such as $-\frac{a}{2}r^2 sin^2\theta$ in the gravitational potential of the Earth.

4.1 Killing vectors of the Kerr and Schwarzchild spacetimes

As we think that the Kerr spacetime models the gravitational field of the Earth better than the Schwarzschild one, symmetry properties of the Kerr spacetime are of crucial importance, specially the observers corrotates along the integral curves of Killing vector fields. By definition, a Killing vector field is a congruence of curves along which the metric tensor is Lie dragged:

$$\mathcal{L}_{\xi} g_{\mu\nu} = 0 \quad \rightarrow$$

$$\xi_{\mu;\nu} + \xi_{\nu;\mu} = \partial_\nu\xi_\mu + \partial_\mu\xi_\nu - 2\Gamma^{\rho}_{\mu\nu}\xi_\rho = 0 \qquad (27)$$

in which ";" denotes the absolute (or covariant) differentiation. The physical significance of a Killing vector should be stressed. The Killing vectors are responsible for existence of any conserved physical quantity of the system under consideration. In fact, physical quantities are conserved only along Killing vector fields of the spacetime. Suppose the spacetime is foliated by spacelike hypersurfaces Σ_t each of them characterized by some $t(x^\mu) = const.$ scalar field. Also a congruence of curves which intersects the hypersurfaces is assumed. Each curve have some normalized timelike tangent vector (four-velocity) u^μ which satisfies $g_{\mu\nu}u^\mu u^\nu = -1$. The spacetime (2) is said to be stationary if and only if it admits some hypersurface orthogonal Killing vector field u^μ. Any physically meaningful gravitational potential $\Phi(x)$, and consequently the corresponding geoid, should satisfies the relation

$$\Phi(\gamma_(\tau)) = \Phi(\hat{\gamma}_(\hat{\tau})) \qquad (28)$$

in which $\gamma_(\tau)$ and $\hat{\gamma}_(\hat{\tau})$ are two members of the above mentioned congruence (observers) each parametrized by its own proper time. Equation (28) says that the gravitational potential should seem the same from the point of view of arbitrary couple of observers. According to (Ehlers 1993), the condition (28) satisfied if an only if the γ curves be Killing vector fields of the spacetime manifold (Philipp, 2017). Needless to say, this is possible only in stationary spacetimes.

The next step is to find a relationship between Killing vectors and the gravitational potential. As it just saied, for the spacetime to be stationary, the condition $\xi^\mu || u^\mu$ is necessary. For example, for the case of the Schwarzchild geometry $\xi^\mu = \sqrt{-g_{00}}u^\mu$ and

$$g_{\mu\nu}\xi^\nu\xi^\mu = g_{00}. \qquad (29)$$

The equipotential surface is defined by $g_{\mu\nu}\xi^\nu\xi^\mu = const.$. This idea was pointed out for the first time by Hermann Weyl in its most general form. Based on this, the general form of equation (9) reads

$$1 + z = \sqrt{\frac{g_{\mu\nu}(X_2)\xi^\mu(X_2)\xi^\nu(X_2)}{g_{\mu\nu}(X_1)\xi^\mu(X_1)\xi^\nu(X_1)}}. \qquad (30)$$

in which ξ^μ is the Killing vector and $z = \frac{\nu_1 - \nu_2}{\nu_2}$ the redshift. Equation (30) holds if the Killing vector satisfies $\xi^\mu || u^\mu$ where u^μ is the velocity 4-vector of the observer. For more information see (Harvey et al. 2006).

For the **Schwarzchild** spacetime, there are two Killing vectors corresponding to the observers rotating in the directions ϕ and t. This is because the Schwarzchild metric do not explicitly depend on the coordinates ϕ and t. The Killing vectors are demonstrated by $\xi_1 = \partial_t$ and $\xi_2 = \partial_\phi$. Evidently, any linear combinations of the two with constant coefficients is also a Killing vector. Consequently, we choose the following forms for the Killing vectors:

$$\begin{aligned} \xi &= \partial_t \\ \zeta &= \Omega\partial_\phi + \partial_t, \end{aligned} \tag{31}$$

where Ω is some constant having dimension rad/s. To find the corresponding geoid, we simply compute

$$\begin{aligned} \xi^\mu &= (1,0,0,0) \rightarrow g_{\mu\nu}\xi^\mu\xi^\nu = 1 - \frac{2m}{r} = const., \\ \zeta^\mu &= (1,0,0,\Omega) \rightarrow g_{\mu\nu}\zeta^\mu\zeta^\nu = 1 - \frac{2m}{r} - \frac{1}{c^2}\Omega^2 r^2 sin^2\theta = const., \end{aligned} \tag{32}$$

where can also be written as

$$\begin{aligned} -\frac{GM}{r} &\equiv W(r) = const., \\ -\frac{GM}{r} - \frac{1}{2c^2}\Omega^2 r^2 sin^2\theta &\equiv W(r) = const., \end{aligned} \tag{33}$$

The second relation shows the fact that rotative observers which orbits along the integral curves of these Killing vectors see the extra term which contributes to the determination of the geoid while the static ones, the first relation in (33), does not. Equation (33) is in complete agreement with the definition of the geoid in classical geodesy.

For the **Kerr** spacetime we do as follows. Since the Kerr spacetime is a rotative black hole its application as model of Earth's gravitational field does not probably suitable. However, it also has important properties which made it a good choice for such a modeling, specially at the weak field limit. Also we limit our analysis to the regions below the radius of the event horizon or more specifically below the inner ergosphere.

4.2 The weak field limit

The metric of the Kerr spactime in the Boyer-Lindquist coordinates is as follows:

$$\begin{aligned} ds^2 = \ & \frac{\Delta - a^2 \sin^2\theta}{\rho^2}\, c^2 dt^2 + \frac{4\, m\, a}{\rho^2}\, r \sin^2\theta\, d\phi\, dt - \frac{\rho^2}{\Delta}\, dr^2 \\ & -\rho^2\, d\theta^2 - \frac{A \sin^2\theta}{\rho^2}\, d\phi^2, \\ & \Delta = r^2 - 2\, m\, r + a^2, \\ & \rho^2 = r^2 + a^2 \cos^2\theta,\ A = (r^2 + a^2)^2 - a^2\, \Delta \sin^2\theta. \end{aligned} \tag{34}$$

where a is the angular momentum per unit mass of the source and m the gravitational mass. Two Killing vectors $\xi = \partial_t,\ \zeta = \Theta_0\partial_\phi + \partial_t$ are also the case here in which Θ_0 is some constant. The rotation due to angular momentum a is responsible for the famous dragging of frames known as Lens-Thirring effect. To give some insight into the case of the Earth, we go through the slowly rotating regime of the Kerr spacetime characterized by condition $a << m$. Accordingly, after some expansion around small parameter $\frac{a}{m} << 1$, the metric reads

$$\begin{aligned} ds^2 = -(1 - \tfrac{2m}{r})\, c^2 dt^2 + (1 - \tfrac{2m}{r})^{-1} dr^2 - \tfrac{4ma}{r} sin^2\theta\ dt d\phi + \\ r^2 d\Omega^2 \end{aligned} \tag{35}$$

where we have kept the terms only up to $O(a^2)$. Therefore, we find Killing vectors and the corresponding geoid as follows:

$$\begin{aligned} g_{\mu\nu}\xi^\mu\xi^\nu &= 1 - \frac{2m}{r} = const. \\ g_{\mu\nu}\xi^\mu\xi^\nu &= 1 - \frac{2m}{r} - \frac{\Theta_0^2}{c^2} r^2 sin^2\theta + 4\frac{\Theta_0}{c^2}\frac{ma}{r} sin^2\theta = const. \end{aligned} \tag{36}$$

The first equation in (35) is the gravitational potential seen by a static observer fixed in some point in space while the second one seen by a stationary observer. As is well-known, static observers in a rotative spacetime do not see the events simultaneously hence are irrelevant to our purpose. For the second relation in (36) an interesting point needs to be addressed. It can be rewritten as

$$-\frac{GM}{r} - \Theta_0(\frac{\Theta_0}{2} - \frac{2ma}{r^3}) r^2 sin^2\theta = const. \tag{37}$$

Compared to the case of the Schwarzchild spacetime, an extra term

$$\omega =: \frac{2ma}{r^3}, \tag{38}$$

has been appeared which is the angular velocity of zero angular momentum observers (ZAMOs) as seen by observers at the infinity, i.e. staying at $r \rightarrow \infty$. In brief, the effect of frame dragging should be taken into account in calculations of the gravitational potential in classical physical geodesy.

5. Conclusion

Specific contributions of the general theory of relativity to the definition of the concept of geoid analyzed. It a static spacetime, i.e. the Schwarzschild, the Killing observers contribute the gravitational potential through an extra term which plays the role of centrifugal potential traditionally added to the potential in classical geodesy. At the next step, it supposed that the gravitational field of the Earth can be modeled by the Kerr spacetime and found the corresponding modifications to the gravitational potential. The inherent angular momentum of the Kerr spacetime shows itself through a term most likely to that of the angular velocity of the ZAMO observers. The possiblity to cancel the dragging effect of the frames was shown.

Further study on the impact of using a rotative spacetime to describe the geoid and other important subjects in classical geodesy may include computation of zonal harmonics and finding the corrections induced by relativistic effects. Such work is under way by the author. Another important development may concern the notion of clock synchronization in a rotative spacetime which has significant importance in interferometry.

Acknowledgements

The author would like to thank University of Tehran for supporting this research under the grant No. 30102/1/01.

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